Hovey, Mark

Additive closed symmetric monoidal structures on $R$-modules. (English)
http://dx.doi.org/10.1016/j.jpaa.2010.06.024
http://www.sciencedirect.com/science/journal/00224049

In this well-written paper, the following question is studied: for which associative rings $R$ the category of $R$-modules admits a structure of a closed symmetric monoidal category, the monoidal product being an additive functor in both variables. In particular, the rings $R$ are found for which there are exactly 0, 1, and 7 such structures.

Pasha Zusmanovich (Tallinn)

Keywords: closed symmetric monoidal category

Classification:
*18D10 Monoidal categories
16D90 Module categories (assoc. rings and algebras)

Kovacevic, Domagoj

Real forms of dual pairs $\mathfrak{g}_2 \times \mathfrak{h}$ in $\mathfrak{g}$ of type $E_6$, $E_7$ and $E_8$. (English)
http://www.heldermann.de/JLT/JLT21/JLT212/jlt21017.htm
http://www.heldermann.de/JLT/jltcover.htm
http://www.emis.de/journals/JLT/

A dual pair in a Lie algebra $\mathfrak{g}$ is a direct sum $\mathfrak{a} \oplus \mathfrak{b}$ of subalgebras $\mathfrak{a}$, $\mathfrak{b}$ of $\mathfrak{g}$ centralizing each other. The author describes real forms of dual pairs in classical Lie algebras of types $E_6$, $E_7$, and $E_8$ (with one of the subalgebras being of type $A_2$, $C_3$, and $F_4$ respectively). The main tool is embedding of $sl(2)$ into various subalgebras of $\mathfrak{g}$.

Pasha Zusmanovich (Tallinn)

Keywords: dual pairs; real forms

Classification:
*17B20 Simple and semisimple Lie algebras
17B25 Exceptional Lie algebras

Lee, Dong-II

Monomial bases for some irreducible $\mathfrak{g}_2$-modules. (English)
J. Algebra Appl. 9, No. 5, 705-715 (2010). ISSN 0219-4988
Basing on known earlier Gröbner–Shirshov base of the “negative part” of the universal enveloping algebra of the simple Lie algebra of type $G_2$, the author computes Gröbner–Shirshov bases of irreducible modules over this algebra.

Pasha Zusmanovich (Tallinn)

Keywords : Gröbner-Shirshov base

Classification :

*17B10 Representations of Lie algebras, algebraic theory
13P10 Polynomial ideals, Groebner bases
16Z05 Computational aspects of associative rings
17-08 Computational methods
17B25 Exceptional Lie algebras
17B35 Universal enveloping algebras (Lie algebras)

Zbl 1210.17028

Krasil’shchik, Iosif

Algebraic theories of brackets and related (co)homologies. (English)
Acta Appl. Math. 109, No. 1, 137-150 (2010). ISSN 0167-8019; ISSN 1572-9036

In this expository paper, summarizing many earlier works of the author, he develops a variant of the general algebraic theory of Frölicher–Schouten–Nijenhuis brackets in the category of modules over a commutative algebra, and make connections with the Poisson cohomology and commuting Poisson structures.

Pasha Zusmanovich (Tallinn)

Keywords : Frölicher–Nijenhuis brackets; Schouten–Nijenhuis brackets; Poisson algebras; Poisson cohomology

Classification :

*17B63 Poisson algebras
13D99 Homological methods (commutative rings)
58J10 Differential complexes
13N99 Differential algebra
17B56 Cohomology of Lie algebras
18G60 Other (co)homologies
53C05 Connections, general theory
Replacing the exterior superalgebra in the usual Chevalley–Eilenberg complex computing the (co)homology of Lie superalgebras by the superalgebra of divided powers (what involves one more dimension – the shearing parameter), the authors introduce a notion of divided power (co)homology of Lie superalgebras.

This notion is especially useful in characteristic 2: the usual interpretation of relations of a Lie (super)algebra as the second homology with coefficients in the trivial module does not work in this case, as the relations of the form $x^2 = 0$ for $x$ odd are not taken into account. Divided power homology fixes this deficiency.

On this way, the authors give presentations of finite-dimensional Lie superalgebras with indecomposable Cartan matrix in characteristic 2. A more complete version of the paper – including an analogous treatment in characteristic $p > 2$ (where, however, the divided power cohomology is not essential and can be replaced by the usual cohomology) – is available as arXiv:0911.0243. In $p = 2$, unlike in higher characteristic, non-Serre relations appear even between Chevalley generators.

The proofs are largely given by reference to the Mathematica-based SuperLie package written by one of the authors (Grozman).

The paper also features a thorough discussion of peculiarities of symmetric and anti-symmetric bilinear forms and associated Lie algebras in characteristic 2, due to another author (Lebedev).

Pasha Zusmanovich (Tallinn)

Keywords: characteristic 2; presentation of a Lie superalgebra; cohomology

Classification:

- 17B56 Cohomology of Lie algebras
- 17B50 Modular Lie algebras
- 17-04 Machine computation, programs (nonassoc. rings and algebras)
- 17B22
- 17B55 Homological methods in theory of Lie algebras
- 17B60 Lie rings associated with other structures
- 15A63 Bilinear forms, etc.

Zbl pre05758255

Wang, Song; Zhu, Linsheng

Non-degenerate invariant bilinear forms on Lie color algebras. (English)

Algebra Colloq. 17, No. 3, 365-374 (2010). ISSN 1005-3867
There are many papers dedicated to the study of Lie algebras with a nondegenerate symmetric invariant bilinear form (so-called quadratic Lie algebras), starting from a remarkable paper by A. Medina and Ph. Revoy [Ann. Sci. Éc. Norm. Supér. (4) 18, 553–561 (1985; Zbl 0592.17006)], where the relationship between symmetric invariant bilinear forms and 2-cocycles was noted, a notion of double extension was introduced, and, using it, quadratic Lie algebras were classified. These observations, notions and results were extended to the super case by H. Benamor and S. Benayadi [Commun. Algebra 27, 67–88 (1999; Zbl 0943.17004)]. Here the authors extend it further, in a straightforward manner, to the color case.

Pasha Zusmanovich (Tallinn)

Keywords: invariant symmetric bilinear form; double extension; 2-cocycle

Classification:

*17B75 Color Lie (super)algebras
17B40 Automorphisms and other operators on Lie algebras

Zbl 1200.17010

Galaev, Anton S.

http://dx.doi.org/10.1016/j.difgeo.2009.09.001
http://www.sciencedirect.com/science/journal/09262245

A Lie subalgebra $L$ of $gl(V)$, where $V$ is a finite-dimensional complex linear space, is called a skew-Berger algebra, if it coincides with the space $R(L)$, the linear span of elements of the form $R(x,y)z$, where $R$ is a linear map $S^2(V) \to L$ satisfying the Bianchi-type condition $R(x,y)z + R(z,x)y + R(y,z)x = 0$ for all $x, y, z \in V$. Berger superalgebras (not skew ones!) are defined in a similar super setting, what involves a linear superspace $V$, a variant of super-Bianchi identity and a super skew-symmetric product instead of the symmetric one. This way, a Berger superalgebra on a purely odd superspace $V$ becomes a skew-Berger algebra on $\Pi(V)$, where $\Pi$ is a parity changing functor.

In this paper, irreducible complex skew-Berger algebras are classified. The author employs an exact sequence connecting the first and second Cartan prolongations of $L$, the second Spencer cohomology of $L$, and $R(L)$, and skillful case-by-case considerations involving simple Lie algebras and superalgebras and their representations, with occasional appeal to computer calculations.

The motivation comes from differential geometry – a classification problem of holonomy algebras of connections on supermanifolds.

Pasha Zusmanovich (Tallinn)
About the cohomology of the Lie superalgebra of vector fields on $\mathbb{R}^{n|n}$. (English)
Commun. Algebra 37, No. 8, 2679-2687 (2009). ISSN 0092-7872; ISSN 1532-4125
http://dx.doi.org/10.1080/00927870902747530
http://www.tandfonline.com/loi/lagb20

The first cohomology of the Lie superalgebra of smooth vector fields on the supermanifold $\mathbb{R}^{n|n}$, with coefficients in the module of smooth differential forms on $\mathbb{R}^{n|n}$, is computed.

Pasha Zusmanovich (Reykjavik)

Diamonds of finite type in thin Lie algebras. (English)
J. Lie Theory 19, No. 1, 185-207 (2009). ISSN 0949-5932
http://www.heldermann.de/JLT/JLT19/JLT191/jlt19008.htm
http://www.heldermann.de/JLT/jltcover.htm
http://www.emis.de/journals/JLT/

Thin Lie algebras are infinite-dimensional $\mathbb{N}$-graded modular Lie algebras with additional technical conditions, all whose homogeneous components are 1- or 2-dimensional. Components of dimension 2 are imaginatively called diamonds, and the question of possible distribution of diamonds across the natural numbers line, is the main question considered in this paper. It is proved that if this distribution follows a specific pattern,
then the thin Lie algebra is a loop algebra of a Block algebra, and a strategy for a more general classification is outlined.

The motivation comes from the theory of pro-$p$-groups, and the paper features a very nicely written survey of this and related questions.

Pasha Zusmanovich (Reykjavik)

Keywords: thin Lie algebras; graded Lie algebras; diamonds; loop algebras

Classification:

- 17B70 Graded Lie algebras
- 17B50 Modular Lie algebras

Zbl 1185.17018

Viviani, Filippo

Infinitesimal deformations of restricted simple Lie algebras. II. (English) J. Pure Appl. Algebra 213, No. 9, 1702-1721 (2009). ISSN 0022-4049

http://dx.doi.org/10.1016/j.jpaa.2009.01.012

http://www.sciencedirect.com/science/journal/00224049

In this paper, the second cohomology with coefficients in the adjoint module of finite-dimensional simple restricted modular Lie algebras of Cartan type of contact ($K_n$) and Hamiltonian ($H_n$) series, is computed (the characteristic of the ground field is assumed to be $> 3$). According to the classical deformation theory, this provides the first step in determining their deformations.

The paper continues the first one in the series [J. Algebra 320, No. 12, 4102–4131 (2008; Zbl 1221.17021)] where the same task is performed for the general ($W_n$) and special ($S_n$) types.

The main results of this paper, along with methods for computations for similar cohomology, were known to A. Dzhumadil’daev since the beginning of the 1980s [his Ph.D. and D.Sc. theses, Moscow State Univ. 1981, 1988]. Unfortunately, only a small part of it, in a somewhat cryptic form and lacking the full proofs, appeared in the mainstream literature [Sov. Math., Dokl. 23, 398–402 (1981); translation from Dokl. Akad. Nauk SSSR 257, 1044–1048 (1981; Zbl 0467.17010). In the paper under review, any reference to this previous work is absent. In the proof, the author utilizes results about cohomology of truncated coinduced modules over modular Lie algebras [Math. Z. 206, No. 1, 153–168 (1991; Zbl 0727.17010)], and the second cohomology with coefficients in the trivial module of Lie algebras of Cartan type [Algebras Groups Geom. 3, 431–455 (1986; Zbl 0621.17012)], due to R. Farnsteiner and H. Strade. It should be noted that these results were also obtained independently (and earlier) by A. Dzhumadil’daev [Mat. Sb. 180, No. 4, 456–468 (1989; Zbl 0691.17008) and Funkt. Anal. Appl. 18, 331–332 (1984); translation from Funkts. Anal. Prilozh. 18, No. 4, 77–78 (1984; Zbl 0569.17005), respectively].

Another intermediate result of this paper – genuinely new and interesting for its own sake – is the computation of the third cohomology with trivial coefficients of the Hamiltonian Lie algebras.

Pasha Zusmanovich (Reykjavik)
Keywords: modular Lie algebras of Cartan type; second cohomology; third cohomology with trivial coefficients of Hamiltonian Lie algebras

Classification:
*17B50 Modular Lie algebras
17B56 Cohomology of Lie algebras

Zbl pre05610489

Bergh, Petter Andreas; Erdmann, Karin

The Avrunin-Scott theorem for quantum complete intersections. (English)
http://dx.doi.org/10.1016/j.jalgebra.2008.12.019
http://www.sciencedirect.com/science/journal/00218693

In [“Quillen stratification for modules”, Invent. Math. 66, 277–286 (1982; Zbl 0489.20042)], G. S. Avrunin and L. L. Scott proved that the support variety of a module over a group algebra $K[\mathbb{Z}/\mathbb{Z}_p^n]$ of an elementary abelian group, is isomorphic to its rank variety. Here, the authors prove a similar result for modules over a quantum complete intersection – a quantum analog of $K[\mathbb{Z}/\mathbb{Z}_p^n]$.

Pasha Zusmanovich (Tallinn)

Keywords: rank varieties; support varieties; quantum complete intersections

Classification:
*16E40 Homology and cohomology theories for assoc. rings
14M99 Special varieties
16S38 Rings arising from non-commutative algebraic geometry

Zbl 1225.17009

Ovsienko, V.

Lie antialgebras: cohomology and representations. (English)
http://dx.doi.org/10.1063/1.3043862

Lie antialgebras is a new class of algebras, introduced by the author in [J. Algebra 325, No. 1, 216–247 (2011; Zbl 05869014); available in preprint form as early as 2007]. They are supercommutative algebras whose multiplication, denoted imaginatively by $\cdot, \cdot,$
satisfies the following additional identities:

\[
\begin{align*}
[x_1, x_2, x_3] & = [x_1, x_2, x_3] \\
[x_1, x_2, y] & = \frac{1}{2} [x_1, x_2, y] \\
[x, y_1, y_2] & = [x, y_1, y_2] + [x, y_2] \\
[y_1, y_2, y_3] & = [y_1, y_2, y_3] + [y_2, y_3, y_1] + [y_3, y_1, y_2] = 0,
\end{align*}
\]

where \(x\)'s are even elements and \(y\)'s are odd ones.

An example: a “conformal Lie antialgebra” \(\mathcal{KA}(1)\) whose even part is spanned by elements \(\{e_n | n \in \mathbb{Z}\}\), the odd part is spanned by elements \(\{\ell_n | n \in \mathbb{Z} + \frac{1}{2}\}\), and the multiplication table is as follows:

\[
\begin{align*}
[e_n, e_m] & = e_{n+m} \\
[e_n, \ell_m] & = \frac{1}{2} \ell_{n+m} \\
[\ell_n, \ell_m] & = \frac{1}{2} (n - m) e_{n+m}.
\end{align*}
\]

The derivation algebra of \(\mathcal{KA}(1)\) is the famous Neveu–Schwarz Lie superalgebra.

Another example: a simple Lie antialgebra \(\text{asl}(2)\) with 1-dimensional even part \(\langle \epsilon \rangle\), a 2-dimensional odd part \(\langle a, b \rangle\), and multiplication table:

\[
\begin{align*}
[\epsilon, \epsilon] & = \epsilon, \quad [\epsilon, a] = \frac{1}{2} a, \quad [\epsilon, b] = \frac{1}{2} b, \quad [a, b] = \frac{1}{2} \epsilon.
\end{align*}
\]

The derivation algebra of \(\text{asl}(2)\) is the classical simple Lie superalgebra \(\text{osp}(1|2)\).

The paper under review is a survey, without proofs and technical details, of results obtained by the author and his collaborators, and elaborated in several other papers. Two very convincing arguments are given why Lie antialgebras might be interesting: the first comes from the symplectic geometry: the space of \(\text{Osp}(1|2)\)-invariant bivector fields on \(\mathbb{R}^2\) is spanned by two vector fields, even and odd ones, the odd one being seemingly overlooked till recently. This odd vector field defines a bilinear operation on the space of functions on \(\mathbb{R}^{2|1}\) which forms an algebra isomorphic to \(\text{asl}(2)\).

The second argument is of algebraic nature: the Massey bracket on the space of bilinear maps on a vector space which is decomposed into the direct sum of two subspaces, being restricted to one of the direct summands, forms, under some technical assumptions, a Lie antialgebra.

The topics covered in this survey are: relation to Lie superalgebras, classification of finite-dimensional simple Lie antialgebras (there is only one, \(\text{asl}(2)\)), and beginnings of representation and cohomology theories.

**Pasha Zusmanovich (Tallinn)**

**Keywords**: Lie antialgebras; conformal algebras; superalgebras; representations; cohomology; symplectic geometry

**Classification**:

* 17A70 Superalgebras
  17A36 Automorphisms and other operators
  17A60 Structure theory of general nonassociative rings and algebras
  17B10 Representations of Lie algebras, algebraic theory
Viviani, Filippo

Infinitesimal deformations of restricted simple Lie algebras. I. (English)
J. Algebra 320, No. 12, 4102-4131 (2008). ISSN 0021-8693
http://dx.doi.org/10.1016/j.jalgebra.2008.08.022
http://www.sciencedirect.com/science/journal/00218693

The second cohomology group with coefficients in the adjoint module of restricted simple Lie algebras of the general ($W_n$) and special ($S_n$) types is computed. This work was done earlier by A. Dzhumadil’daev, but the detailed proofs were not published. The second part of the paper, with a more detailed review [see, J. Pure Appl. Algebra 213, No. 9, 1702–1721 (2009; Zbl 1185.17018)] performs the same task in the remaining cases of contact and Hamiltonian types.

Pasha Zusmanovich (Tallinn)

Keywords: modular Lie algebras of Cartan type; second cohomology

Classification:

- 17B50 Modular Lie algebras
- 17B56 Cohomology of Lie algebras

Takamura, Masashi

The relative cohomology of formal contact vector fields with respect to formal Poisson vector fields. (English)
http://dx.doi.org/10.2969/jmsj/06010117
http://projecteuclid.org/jmsj
http://www.jstage.jst.go.jp/browse/jmath/

Refining the method of Gelfand–Fuks [see D. Fuks, Cohomology of Infinite-Dimensional Lie Algebras, Moskva: Nauka (1984; Zbl 0592.17011)], the author establishes that the cohomology with trivial coefficients of the Lie algebra $K$ of formal contact vector fields relative to its subalgebra of Poisson formal vector fields is trivial. He also recovers the previously known fact that the absolute cohomology of $K$ vanishes in the big enough degree.

Pasha Zusmanovich (Tallinn)

Keywords: Lie algebra of formal contact vector fields; Gelfand-Fuks cohomology
This is another incremental step in ongoing efforts to classify what is amenable to classification in the realm of nilpotent Lie algebras, i.e. filiform and close to them algebras.

An $n$-dimensional Lie algebra is called quasi-filiform if its lower central series terminates exactly at the $(n - 2)$th step. The length of such algebra is the maximum of lengths of its N-gradations.

Here, graded quasi-filiform Lie algebras of finite dimension $n \geq 15$ and length $n - 1$, are classified. There are 22 types of such algebras, some of them constitute parametric families.

The paper consists, essentially, of long and cumbersome computations performed with the aid of Mathematica.

Pasha Zusmanovich (Reykjavik)

Keywords: quasi-filiform Lie algebra

Classification:

- 17B30 Solvable, nilpotent Lie algebras
- 04 Machine computation, programs (nonassoc. rings and algebras)
- 17B70 Graded Lie algebras

The Iwasawa algebra is the completion of a group algebra of a compact $p$-adic analytic group with respect to open normal subgroups.

The authors prove that for a Lie group of a split semisimple Lie algebra over a field of $p$-adic numbers, the corresponding Iwasawa algebra $A$ does not have non-trivial two-sided...
ideals $I$ such that the canonical map $I \rightarrow \text{Hom}_A(\text{Hom}_A(I, A), A)$ is an isomorphism. A simple yet elegant technique of interplay between Lie algebras defined over $p$-adic and prime fields, and passing from a Lie algebra to another Lie algebra with desired $p$-torsion properties, is used.

*Pasha Zusmanovich (Reykjavik)*

**Keywords**: Iwasawa algebras; reflexive ideals; powerful Lie algebras

**Classification**:

- 16S34 Group rings (assoc. rings)
- 16D25 2-sided ideals (assoc. rings and algebras)
- 16W60 Filtrations and valuations, etc. (assoc. rings and algebras)
- 17B20 Simple and semisimple Lie algebras
- 17B50 Modular Lie algebras
- 20C07 Group rings of infinite groups and their modules (group theory)
- 20G25 Linear algebraic groups over local fields and their integers
- 11R23 Iwasawa theory
- 20E18 Profinite groups
- 22E50 Repres. of Lie and linear algebraic groups over local fields

**Zbl 1144.17014**

*Fialowski, Alice; Wagemann, Friedrich*

**Cohomology and deformations of the infinite-dimensional filiform Lie algebra $m_2$, (English)**

J. Algebra 319, No. 12, 5125-5143 (2008). ISSN 0021-8693

http://dx.doi.org/10.1016/j.jalgebra.2008.02.035

http://www.sciencedirect.com/science/journal/00218693

The authors describe low-dimensional cohomology with coefficients in the adjoint module and their Massey products, and deformations of the infinite-dimensional Lie algebra given by the basis $\{e_1, e_2, \ldots\}$ with multiplication table $[e_1, e_i] = e_{i+1}$, $i \geq 2$, $[e_2, e_i] = e_{i+2}$, $i \geq 3$ (all other products between basic elements vanish). The method consists of more or less direct computations.

This Lie algebra, together with other two, exhaust all 2-generated $N$-graded Lie algebras with 1-dimensional graded components. Low-dimensional cohomology and deformations of the other two Lie algebras were studied by the author(s) earlier.

*Pasha Zusmanovich (Reykjavik)*

**Keywords**: deformation; Massey product

**Classification**:

- 17B56 Cohomology of Lie algebras
- 17B65 Infinite-dimensional Lie algebras
- 17B70 Graded Lie algebras
Courant brackets on noncommutative algebras and omni-Lie algebras. (English)
http://dx.doi.org/10.3836/tjm/1184963659
euclid:tjm/1184963659
http://projecteuclid.org/tjm


In this paper, a similarly-looking bracket is defined in a purely algebraic situation, on the space $E(A) = H^1(A, A) \oplus H_1(A, A)$, where $H^1$, resp. $H_1$, denotes the first Hochschild cohomology, resp. homology of an associative algebra $A$. A canonical pairing between $H^1(A, A)$ and $H_1(A, A)$ induces a bilinear form on $E(A)$, and the kernel of this form happens to be an ideal with respect to the Courant bracket. The quotient of $E(A)$ by this ideal, with the induced Courant bracket on it, is called a Dirac structure on $A$.

In the case $A = V \oplus K_1$, where multiplication on $V$ is trivial, the Dirac structure reduces to the $A$. Weinstein’s omni-Lie algebra defined on the space $gl(V) \oplus V$ [RIMS Kokyuroku 1176, 95–102 (2000; Zbl 1058.58503)].

It is proved that Dirac structures on any unital associative algebra $A$ and the algebra of matrices of arbitrary size over $A$ are isomorphic.

Dirac structures are also connected with commutative and noncommutative Poisson structures.

Pasha Zusmanovich (Tallinn)

Keywords : Courant bracket; omni-Lie algebra

Classification :

*17A99 General nonassociative rings
16E40 Homology and cohomology theories for assoc. rings
17B60 Lie rings associated with other structures
17B63 Poisson algebras

Zbl 1177.17010

Panyushev, Dmitri I.

On the coadjoint representation of $\mathbb{Z}_2$-contractions of reductive Lie algebras. (English)
http://dx.doi.org/10.1016/j.aim.2006.12.011
http://www.sciencedirect.com/science/journal/00018708

This paper is devoted to Lie algebras, defined over an algebraically closed field $K$
of characteristic zero, represented as the semidirect sum $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$, where $\mathfrak{g}_0$ is a finite-dimensional simple Lie algebra and $\mathfrak{g}_1$ is a $\mathfrak{g}_0$-module. These algebras could be considered as contractions of a reductive Lie algebra with a symmetric decomposition $\mathfrak{g}_0 \oplus \mathfrak{g}_1$, and share many properties with the latters.

The author conjectures that the algebra of invariants of the coadjoint representation of such semidirect products is polynomial, and proves this in some particular cases.

Another particular case, considered earlier by several authors, is $\mathfrak{g}_0 \simeq \mathfrak{g}_1$, in which case $\mathfrak{g} \simeq \mathfrak{g}_0 \otimes K[t]/(t^2)$.

Pasha Zusmanovich (Reykjavik)

Keywords: algebra of invariants; contraction; semidirect product; Takiff algebra; current Lie algebra

Classification:

*17B10 Representations of Lie algebras, algebraic theory
14R20 Group actions on affine varieties
17B20 Simple and semisimple Lie algebras

Zbl 1173.17010

Penkov, Ivan; Zuckerman, Gregg

Construction of generalized Harish-Chandra modules with arbitrary minimal \(\mathfrak{t}\)-type. (English)


http://dx.doi.org/10.4153/CMB-2007-059-5

http://www.cms.math.ca/cmb/

This paper is a part of the project whose ultimate goal is the classification of simple generalized Harish-Chandra modules over a semisimple finite-dimensional complex Lie algebra $\mathfrak{g}$ (i.e. infinite-dimensional modules having finite-dimensional isotypic components as a module over some reductive subalgebra $\mathfrak{t}$).

From the abstract: “For any simple finite dimensional \(\mathfrak{t}\)-module $V$, we construct simple $(\mathfrak{g}, \mathfrak{t})$-modules $M$ with finite dimensional $\mathfrak{t}$-isotypic components such that $V$ is a $\mathfrak{t}$-submodule of $M$ and the Vogan norm of any simple $\mathfrak{t}$-submodule $V' \subset M$, $V' \not\simeq V$, is greater than the Vogan norm of $V$”.

Pasha Zusmanovich (Reykjavik)

Keywords: generalized Harish-Chandra module; isotypic component; Vogan norm

Classification:

*17B10 Representations of Lie algebras, algebraic theory
17B55 Homological methods in theory of Lie algebras

Zbl 1171.17008

Conley, Charles H.; Martin, Christiane

Annihilators of tensor density modules. (English)

J. Algebra 312, No. 1, 495-526 (2007). ISSN 0021-8693
This paper concerns the representation theory of the ubiquitous Lie algebras of vector fields on the line $\text{Vec}(\mathbb{R})$ and on the circle $\text{Vec}(S^1)$ (also known as one-sided and two-sided Witt algebras, respectively). First the authors get the results in the one-sided case, and then derive from it the two-sided one.

From the (slightly edited) abstract: “We describe the two-sided ideals in the universal enveloping algebras of $\text{Vec}(\mathbb{R})$ and $\text{Vec}(S^1)$ which annihilate the tensor density modules. Both of these Lie algebras contain the projective subalgebra, a copy of $\mathfrak{sl}_2$. The restrictions of the tensor density modules to this subalgebra are duals of Verma modules (of $\mathfrak{sl}_2$) for $\text{Vec}(\mathbb{R})$ and principal series modules (of $\mathfrak{sl}_2$) for $\text{Vec}(S^1)$. Thus our results are related to the well-known theorem of Duflo describing the annihilating ideals of Verma modules of reductive Lie algebras. We find that, in general, the annihilator of a tensor density module is generated by the Duflo generator of its annihilator over $\mathfrak{sl}_2$ (the Casimir operator minus a scalar) together with one other generator, a cubic element of the universal enveloping algebra of $\text{Vec}(\mathbb{R})$.”

The authors also make some remarks and pose questions about modules of differential operators between tensor density modules, and what happens in the multi-dimensional case of $\text{Vec}(\mathbb{R}^n)$.

**Pasha Zusmanovich** (Reykjavik)

Keywords: Lie algebras of vector fields; Witt algebras; tensor density modules

Classification:

*17B66* Lie algebras of vector fields and related algebras
*17B10* Representations of Lie algebras, algebraic theory
*17B20* Simple and semisimple Lie algebras
*17B35* Universal enveloping algebras (Lie algebras)
*17B68* Virasoro and related algebras

Zbl 1161.37047

**Ovsienko, V.; Roger, C.**

**Looped cotangent Virasoro algebra and nonlinear integrable systems in dimension 2 + 1.** (English)


http://dx.doi.org/10.1007/s00220-007-0237-z

http://www.springerlink.com/content/100467/

http://ProjectEuclid.org/cmp

How to generalize the famous Virasoro algebra to the case of several variables, is an interesting question which was considered many times in the literature under different viewpoints. In this paper, the authors are motivated by desire to obtain various versions of the Kadomtsev–Petviashvili equation as appropriate Euler equations.
To this aim, they propose a 2-variable generalization as a central extension of the loop algebra over Lie algebras closely related to the initial Virasoro algebra.

It is known that all central extensions of the loop algebra of the general form $L \otimes A$ are reduced to those obtained from the central extensions of the Lie algebra $L$ (dubbed by authors as “Virasoro type”) and those obtained from symmetric invariant bilinear forms on $L$ and the first-order cyclic cohomology of an associative commutative algebra $A$ (dubbed by authors as “Kac-Moody type”).

As the Kadomtsev–Petviashvili equations contain two different central charges, one of them has to be of the Kac-Moody type.

First the authors consider the loop algebra over the Virasoro algebra itself, but this algebra has central extensions of the Virasoro type only. Then they consider the loop algebra over the semidirect product of the Virasoro algebra and its dual. This loop algebra has central extensions both of Virasoro and Kac-Moody types.

Then they compute several Euler equations corresponding to the latter Lie algebra. These Euler equations turn out to be closely related to the Kadomtsev–Petviashvili equations.

Superization of this situation is also considered, and the corresponding bi-hamiltonian hierarchies are computed.

Note that at least algebraic counterpart of the question raised by the authors in the appendix about the second cohomology of the Lie algebra of vector fields collinear to a given vector field, is solved by S. Skryabin in [Lobachevskii J. Math. 14, 69–107 (2004; Zbl 1044.17012)].

*37K10 Completely integrable systems etc.
17B56 Cohomology of Lie algebras
17B66 Lie algebras of vector fields and related algebras
17B68 Virasoro and related algebras
37K30 Relations with algebraic structures
37K65 Hamiltonian systems on groups of diffeomorphisms etc.
Keywords: root system

Classification:
*17B20 Simple and semisimple Lie algebras

Zbl 1140.17016

Lebedev, A.

On the Bott-Borel-Weil theorem. (English. Russian original)
http://dx.doi.org/10.1134/S0001434607030169
http://www.springerlink.com/content/106483/

The classical Bott-Borel-Weil theorem describes the dimension of the cohomology of the nilradical of the Borel subalgebra of the classical simple Lie algebra $\mathfrak{g}$ with coefficients in an irreducible $\mathfrak{g}$-module, in terms of the corresponding Weyl group. However, the description of the cohomology itself, though known in many particular cases, remains unknown in general.

Here the author considers the case of the trivial module. He determines the explicit basis of the corresponding cohomology and describes, up to sign, its commutative superalgebra structure.

The translation of the paper, as it unfortunately often happens, is poor. However, this should not pose any problem to a specialist.

Pasha Zusmanovich (Reykjavik)

Keywords: Bott-Borel-Weil; cohomology of the nilradical of Borel subalgebra; commutative superalgebra

Classification:
*17B56 Cohomology of Lie algebras
17B20 Simple and semisimple Lie algebras

Zbl 1137.17012

de Graaf, Willem A.

Classification of 6-dimensional nilpotent Lie algebras over fields of characteristic not 2. (English)
http://dx.doi.org/10.1016/j.jalgebra.2006.08.006
http://www.sciencedirect.com/science/journal/00218693

The author performs a painstaking task of comparison of numerous existing classifications of low-dimensional nilpotent Lie algebras, point out errors in them and put the classification on algorithmic ground: for any given nilpotent Lie algebra of dimension not greater than 6, there is a procedure allowing to identify it with an algebra from the classification list. Moreover, the procedure is implemented in GAP.

\textbf{Pasha Zusmanovich (Reykjavik)}  
\textit{Keywords} : nilpotent Lie algebras; low-dimensional Lie algebras; classification; central extension; Gröbner base; GAP  
\textit{Classification} :  
\*17B30 Solvable, nilpotent Lie algebras  
17-08 Computational methods

\textbf{Zbl 1130.17010}  
\textbf{Wu, Yuezhu; Song, Guang’ai; Su, Yucai}  
\textbf{Lie bialgebras of generalized Witt type. II.} (English)  
http://dx.doi.org/10.1080/00927870701247187  
http://www.tandfonline.com/loi/lagb20

Since the pioneering works of Belavin and Drinfeld, people started to be interested in Lie bialgebra structures. For a given Lie algebra \( L \), such structures are in bijective correspondence with cocycles from \( Z^1(L, L \wedge L) \), so one naturally distinguish coboundary and non-coboundary Lie bialgebra structures, depending whether the cohomology class of the corresponding cocycle vanishes or not. In [Quantum deformations of algebras and their representations. Isr. Math. Conf. Proc. 7, 13–24 (1993; Zbl 0852.17013)], A. S. Dzhumadil’daev described non-coboundary Lie bialgebra structures on the Witt and Virasoro algebras, and since then people were rushing to extend these results to plethora of various generalizations of the Witt algebra.

In this paper, the authors deal with another such generalization, too cumbersome to describe here. Their calculations are essentially reduced to computation of the first cohomology of the Witt algebra in the tensor product of various irreducible modules, what in turn can be readily deduced from another result by A. S. Dzhumadil’daev [Z. Phys. C 72, No. 3, 509–517 (1996; Zbl 0935.17012)].

\textbf{Pasha Zusmanovich (Reykjavik)}  
\textit{Keywords} : Witt algebra; Lie bialgebras; coboundary  
\textit{Classification} :  
\*17B62 Lie bialgebras  
17B56 Cohomology of Lie algebras  
17B68 Virasoro and related algebras

\textbf{Zbl 1173.17020}  
\textbf{Hamilton, Alastair}  
\textbf{A super-analogue of Kontsevich’s theorem on graph homology.} (English)  

Pasha Zusmanovich (Reykjavik)

Keywords : Graph complex; Lie superalgebra of symplectic vector fields; homology

Classification :

*17B56 Cohomology of Lie algebras
05C99 Graph theory
17B20 Simple and semisimple Lie algebras
17B55 Homological methods in theory of Lie algebras
17B66 Lie algebras of vector fields and related algebras
18G35 Chain complexes (homological algebra)

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Zbl 1156.17002

Grantcharov, Dimitar

On the structure and characters of weight modules. (English)
Forum Math. 18, No. 6, 933-950 (2006). ISSN 0933-7741; ISSN 1435-5337
http://dx.doi.org/10.1515/FORUM.2006.047
http://www.deGruyter.de/journals/forum/

This paper is devoted to parabolically induced modules (what includes, in particular, simple modules with finite weight multiplicities) over a classical Lie superalgebra of type I (i.e. those admitting a $\mathbb{Z}$-grading concentrated in degrees $-1, 0, 1$) and over a Lie superalgebra of the general Cartan type.

In particular, results about $g_0$-composition series (where $g_0$ is the zero component in the respective superalgebra grading), and character formulas are obtained. The main tool is a certain localization technique introduced by O. Mathieu in [Ann. Inst. Fourier 50, 537-592 (2000; Zbl 0962.17002)].

Pasha Zusmanovich (Reykjavik)

Keywords : simple Lie superalgebra; weight modules; localization; character formula

Classification :

*17B10 Representations of Lie algebras, algebraic theory
17B20 Simple and semisimple Lie algebras
The standard approach to homology theory of Lie algebroids is to define homology via generating operators of low degree (such as flat connection and divergence) for the Schouten bracket. Here the authors propose an alternative approach, based on divergence defined for the so-called “odd-forms”, and interpret in its terms the modular class of Lie algebroids.

The advantage of this approach is that the so defined homology does not depend on the choice of generating operators.

Pasha Zusmanovich (Reykjavik)

Keywords: homology of Lie algebroid; connection; modular class

Classification:

* 17B56 Cohomology of Lie algebras
17B63 Poisson algebras
18G60 Other (co)homologies
53C05 Connections, general theory

This is a shortened version of another paper [A. Fialowski and M. Penkava, Commun. Contemp. Math. 7, No. 2, 145–165 (2005; Zbl 1075.14007)].

Pasha Zusmanovich (Reykjavik)

Keywords: versal deformations; moduli space; low-dimensional Lie algebras

Classification:

* 17B55 Homological methods in theory of Lie algebras
18G55 Nonabelian homotopical algebra
14D15 (Formal) deformations
The second continuous cohomology group with trivial coefficients of the Lie algebra of symplectic vector fields on a compact symplectic manifold, and its subalgebra of Hamiltonian vector fields is studied, by the way of extendibility of the corresponding 2-cocycles.

**Pasha Zusmanovich (Reykjavik)**

**Keywords**: Lie algebra of symplectic vector fields; Lie algebra of Hamiltonian vector fields; central extensions; de Rham cohomology

**Classification**: 
- 17B56 Cohomology of Lie algebras
- 17B66 Lie algebras of vector fields and related algebras
- 58A12 de Rham theory (global analysis)

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The authors prove a super-analog of the celebrated Amitsur-Levitski theorem about vanishing of the standard polynomial for the Lie superalgebra \(osp(1,2n)\), following the celebrated cohomological proof by Kostant. They also outline the reasons why a super-analog of the Amitsur-Levitski theorem unlikely to exists in general (recall in this connection that \(osp(1,2n)\) is the only simple Lie superalgebra whose representations are completely reducible). The proof follows the Kostant approach with some simplifications (which are also applicable in the ordinary Lie-algebraic case).

A shortened version of this article was previously published in [Lett. Math. Phys. 66, 141–155 (2003; Zbl 1051.17010)].

**Pasha Zusmanovich (Reykjavik)**

**Keywords**: Amitsur–Levitsky theorem; Lie superalgebra \(osp(1,2n)\)
As the title suggests, the authors do to Lie superalgebras what Rudakov did to Lie algebras almost four decades ago [A. N. Rudakov, Math. USSR, Izv. 3, 707–722 (1969); translation from Izv. Akad. Nauk SSSR, Ser. Mat. 33, 748–764 (1969; Zbl 0222.17014)]; they compute the group of automorphisms of all simple infinite-dimensional linearly compact Lie superalgebras of characteristic zero. This, via the usual mechanism of Galois cohomology, provides the description of their forms.

Pasha Zusmanovich (Reykjavik)

Keywords: Infinite-dimensional simple Lie superalgebras; automorphisms; Galois cohomology; forms

Classification:

*17B66 Lie algebras of vector fields and related algebras
17B40 Automorphisms and other operators on Lie algebras
17B65 Infinite-dimensional Lie algebras
17B70 Graded Lie algebras

The authors exhibit examples of simple Lie superalgebras of small characteristics as Lie algebras of vector fields in terms of prolongations of a “beginning” part of a graded simple exceptional Lie superalgebra. A similar work for Lie algebras was initiated by P. Grozman and D. Leites in [Lett. Math. Phys. 74, 229–262 (2005; Zbl 1146.17014)]. The paper contains rather provocative parallels and statements and is an interesting read. A preprint version of the paper is available as arXiv:math.RT/0606682.
Pasha Zusmanovich (Reykjavik)

Keywords: exceptional Lie superalgebra; modular Lie superalgebra; small characteristic; prolongation

Classification:

- 17B50 Modular Lie algebras
- 17B25 Exceptional Lie algebras
- 17B70 Graded Lie algebras

Zbl 1188.17015

Chebochko, N.G.

Deformations of classical Lie algebras with homogeneous root system in characteristic two. I. (English)

Sb. Math. 196, No. 9, 1371-1402 (2005); translation from Mat. Sb. 196, No. 9, 125-156 (2005). ISSN 1064-5616

http://dx.doi.org/10.1070/SM2005v196n09ABEH003647


It is widely known that finite-dimensional Lie algebras over a field of small characteristic \( p \) exhibit various “pathologies” absent in the generic positive characteristic case. For example, the classical Lie algebras are known to have zero second degree cohomology in the adjoint module (and thus no nontrivial deformations) if \( p > 3 \), but this is no longer true in small characteristics. In this paper, the author elucidates to which degree this is not true when the ground field \( K \) is of characteristic 2 and the classical Lie algebra has a homogeneous root system (i.e. of type \( A \), \( D \) or \( E \)). And indeed, there are a lot of nontrivial cocycles!

To start with, in characteristic 2 the classical Lie algebras are not simple, but have a nonzero center, quotient by which is a simple algebra. The author considers both types – the original classical algebras (denoted, as usual, by \( X_l \), where \( X \) is one of \( A \), \( D \) or \( E \)), and their simple quotients (denoted by \( \overline{X}_l \)).

Most of the proofs go via case-by-case careful calculations performed by induction of the rank of the algebra, the algebra \( A_2 \) being the basis of induction (the author performs calculations for all the cases – both \( X_l \) and \( \overline{X}_l \) – side-by-side; it seems that the standard tools of homological algebra, such as the Hochschild–Serre spectral sequence and the long exact cohomological sequence associated with the short exact sequence of modules, could be used instead to reduce the former cases to the latter ones). An important role plays an action of the corresponding Chevalley group of the underlying Lie algebra \( L \) on cohomology in question. For example, it turns out that the zero weight subspace under this action vanishes in all cases. The latter is proved via explicit identification of two algebra structures on the tensor product \( L \otimes K[t]/(t^2) \): one is the usual current algebra, and another is its “twisting” by the corresponding cocycle.

Such kind of calculations are important in developing the classification of simple finite-dimensional Lie algebras in characteristic 2: the previous experience suggests that deformations of algebras play an important role, and the second cohomology in question represents “infinitesimal” deformations.
It is interesting to compare this paper with another one, by Sh. Sh. Ibraev and G. A. Turetaeva [Mat. Zh. 7, No. 1, 49–54 (2007)], who aim to solve exactly the same problem. Though the latter paper was submitted 2.5 years after the paper under review, the authors were seemingly (and surprisingly) unaware about it. Their methods are quite different, but the results are the same, except for the case $D_l$ which seems to be just an omission.

Pasha Zusmanovich (Reykjavik)

Keywords: characteristic 2; classical Lie algebras; root system; second cohomology group

Classification:

* 17B56 Cohomology of Lie algebras
  17B20 Simple and semisimple Lie algebras
  17B45 Lie algebras of linear algebraic groups
  17B50 Modular Lie algebras

Zbl 1106.17024

Kornyak, V. V.

Cohomology of restricted Lie algebras of Hamiltonian vector fields: computer analysis. (English. Russian original)
http://dx.doi.org/10.1007/s11086-005-0018-4

The author continues his computer-aided studies of cohomology of Hamiltonian Lie algebras in positive characteristic by means of a non-standard (symmetric) grading.

Pasha Zusmanovich (Reykjavik)

Keywords: cohomology of Hamiltonian Lie algebra

Classification:

* 17B56 Cohomology of Lie algebras
  17B45 Lie algebras of linear algebraic groups
  17B50 Modular Lie algebras

Zbl 1106.17028

Michée, Sébastien; Novitchkov, Gleb

BV-generators and Lie algebroids. (English)
http://dx.doi.org/10.1142/S0129167X05003247
http://www.worldscinet.com/ijm/ijm.shtml

Recall that a Gerstenhaber algebra is a non-negatively graded vector space equipped with compatible associative commutative superalgebra and Lie superalgebra structures,
and a Batalin-Vilkovisky (BV-) algebra is a Gerstenhaber algebra equipped with a linear operator $D$ (called BV-generator) of degree $-1$ such that $D^2 = 0$ and the Lie bracket is expressed in a certain way through $D$ (measuring, roughly, the deviation of $D$ from being a derivation).

The authors give a criterion for a Gerstenhaber algebra to be a BV-algebra. Applying it to a Gerstenhaber algebra associated to a Lie algebroid, they recover a correspondence between BV-generators and flat connections, and obtain a correspondence between Lie algebroid homology with coefficients in a flat connection on a trivial line bundle and cohomology with trivial coefficients.

Pasha Zusmanovich (Reykjavik)

Keywords: Gerstenhaber algebra; BV-algebra; Lie algebroid; flat connection

Classification:

* 17B66 Lie algebras of vector fields and related algebras
* 18G60 Other (co)homologies
* 53C05 Connections, general theory
* 58H05 Pseudogroups on manifolds

Zbl 1106.17006

Serganova, Vera

On representations of Cartan type Lie superalgebras. (English)


Formulas for characters of irreducible highest weight modules over finite-dimensional Lie superalgebras of Cartan type are obtained.

Pasha Zusmanovich (Reykjavik)

Keywords: Lie superalgebra of Cartan type; highest weight module

Classification:

* 17B10 Representations of Lie algebras, algebraic theory
* 17B66 Lie algebras of vector fields and related algebras
* 17B35 Universal enveloping algebras (Lie algebras)
* 17B40 Automorphisms and other operators on Lie algebras

Zbl 1106.17025

Lebedev, Alexei; Leites, Dimitry; Shereshevskii, Ilya

Lie superalgebra structures in $C^\bullet(n; n)$ and $H^\bullet(n, n)$. (English)


The authors are concerned with the cohomology of a maximal nilpotent subalgebra $\mathfrak{n}$ of a classical simple Lie algebra $\mathfrak{g}$ with coefficients in an adjoint module $\mathfrak{n}$. 24

Here the authors again describe explicitly the first cohomology group $H^1(n, n)$ in the general case (though the basic cocycles they provide do not wholly coincide with those of previous authors), and – what is a new result – the Lie superalgebra structure of the whole $H^*(n, n)$ in the cases $g = A_2$ and $G_2$.

The paper also contains a voluminous and interesting discussion of related questions. The preprint version of the paper is available as arXiv:math.KT/0404139.

Pasha Zusmanovich (Reykjavik)

Keywords : maximal nilpotent subalgebra of a classical Lie algebra; cohomology

Classification :
- 17B56 Cohomology of Lie algebras
- 17-04 Machine computation, programs (nonassoc. rings and algebras)
- 17B20 Simple and semisimple Lie algebras
- 17B70 Graded Lie algebras

Zbl 1099.17012

Petit, Toukaiddine

On the strong rigidity of solvable Lie algebras. (English)


This paper is a close exposition of another one [M. Bordemann, A. Makhlouf and T. Petit, J. Algebra 285, 623–648 (2005; Zbl 1099.17009), see also arXiv:math.RA/0211416], where the notion of strong rigidity for finite-dimensional Lie algebras over a field of characteristic 0 is studied. A Lie algebra is said strongly rigid if its universal algebra is rigid (as an associative algebra). Some of the results mentioned below are given with full proofs, and some are just cited.

A strongly rigid Lie algebra is rigid and its second cohomology with trivial coefficients vanishes. If $L^*$ possesses a linear Poisson structure, then $L$ is strongly rigid. A Lie algebra of the form $T \oplus N$, where $T$ is a torus acting on nilradical $N$, $\dim N > 1$, cannot be strongly rigid.

Using these results, and the classification of solvable rigid Lie algebras of small dimension, it is shown that the only solvable strongly rigid Lie algebra of dimension $\leq 6$, is a two-dimensional nonabelian Lie algebra.

The paper is sloppy with regards to language (the reviewer has caught quite a few gallicisms) and typesetting.
Su, Yucai
Classification of quasifinite modules over Lie algebras of matrix differential operators on the circle. (English)
http://dx.doi.org/10.1090/S0002-9939-05-07881-0
http://www.ams.org/proc/

It is proved that an irreducible graded module with finite-dimensional graded components over the central extension of the Lie algebra of $N \times N$-matrix differential operators on the circle is either a highest or lowest weight module, or a module all whose nonzero graded components are 1-dimensional.

Also, indecomposable modules with bounded dimensions of graded components are classified.

This extends the earlier results of the author [Adv. Math. 174, 57–68 (2003; Zbl 1091.17004)] from the case $N = 1$ (he also notes an error in that paper which he corrects in the present one).

Sköldberg, Emil
The homology of Heisenberg Lie algebras over fields of characteristic two. (English)
http://dx.doi.org/10.3318/PRIA.2005.105.2.47
http://www.ria.ie/Publications/Journals/Mathematical-Proceedings/Online-access.aspx

Here the author proposes another interesting approach to this cohomology with trivial coefficients, peculiar to characteristic 2, using the “discrete algebraic Morse theory” developed by him in \([\text{Trans. Am. Math. Soc. 358, No. 1, 115–129 (2006; Zbl 1150.16008)}]\) (the latter paper is required to fully understand the present one, due to free utilization of its notations and definitions; it also contains further homological applications). This machinery resembles, to a certain degree, the “homological perturbation theory” of Gugenheim, Lambe et al.

Preprint version of this paper is available as \texttt{arXiv:math.RT/0312124}.

Pasha Zusmanovich (Ramat Gan)

Keywords : cohomology of Heisenberg Lie algebra; discrete Morse theory; characteristic 2

Classification :
*17B56 Cohomology of Lie algebras
18G35 Chain complexes (homological algebra)

Zbl 1094.17010

Gargoubi, H.; Mathonet, P.; Ovsienko, V.

Symmetries of modules of differential operators. (English)
http://dx.doi.org/10.2991/jnmp.2005.12.3.4
http://www.worldscinet.com/jnmp/jnmp.shtml

Let \(M\) be a smooth manifold, \(F_\lambda(M)\) is the space of tensor densities of degree \(\lambda \in \mathbb{C}\), and \(D^{k}_{\lambda,\mu}(M)\) is the space of \(k\)-th order linear differential operators acting from \(F_\lambda(M)\) to \(F_\mu(M)\).

The authors determine the module of operators from \(D^{k}_{\lambda,\mu}(M)\) invariant under \(\text{Diff}(M)\)-action in cases \(M = S^1\) and \(\mathbb{R}\).

The classification is split in many particular cases depending on the (small) values of \(k\), \(\lambda\) and \(\mu\), and exhibits particular modules with remarkable properties.

Considered as associative operator algebras, these modules are always isomorphic to a direct sum of matrix algebras of 4 particular types.

The case of \(\dim M \geq 2\) was treated in \([P. Mathonet, Commun. Algebra 27, 755–776 (1999; Zbl 0924.17017)]\) and is much simpler.
Pasha Zusmanovich (Ramat Gan)

Keywords: invariant differential operators; module of tensor densities

Classification:
- 17B66 Lie algebras of vector fields and related algebras
- 22E65 Infinite-dimensional Lie groups

Zbl 1080.17010

Hansoul, Sarah; Lecomte, Pierre

Affine representations of Lie algebras and geometric interpretation in the case of smooth manifolds. (English)
http://dx.doi.org/10.1155/IMRN.2005.981
http://imrn.oxfordjournals.org/

The authors systematically develop the notion of affine representation of a Lie algebra, and consider low-dimensional cohomology of Lie algebras naturally associated with it. Then they compute the 0- and 1-dimensional cohomology of the Lie algebra of vector fields on a manifold $M$ with coefficients in some polynomial spaces, what enables to classify affine representations of this Lie algebra in the space of symmetric tensor fields of type $(\frac{1}{2})$ on $M$.

A preprint version of the paper is available as arXiv:math.DG/0307261.

Pasha Zusmanovich (Ramat Gan)

Keywords: affine representation; low dimensional cohomology of a Lie algebra; Lie algebra of vector fields

Classification:
- 17B56 Cohomology of Lie algebras
- 17B10 Representations of Lie algebras, algebraic theory
- 17B66 Lie algebras of vector fields and related algebras
- 53D50 Geometric quantization

Zbl 1062.17014

Bouarroudj, Sofiane

Cohomology of the vector fields Lie algebras on $\mathbb{R}P^1$ acting on bilinear differential operators. (English)
http://dx.doi.org/10.1142/S0219887805000430
http://www.worldscinet.com/ijgmmp/ijgmmp.shtml

This work continues investigations of low-dimensional cohomology of Lie algebras of vector fields in modules of tensor densities and close to these, carried out by the author and other members of V. Ovsienko’s school.
Here the author computes two such cohomology groups: the first cohomology of \( sl(2) \) with coefficients in the module of tensor densities, and the first cohomology of the Lie algebra of smooth vector fields on the projective line relative to its \( sl(2) \) subalgebra in the module of the space of bilinear differential operators on the module of tensor densities.

\textit{Pasha Zusmanovich (Ramat Gan)}

\textit{Keywords} : first cohomology group; Lie algebras of vector fields; module of tensor densities

\textit{Classification} :

\begin{itemize}
  \item 17B56 Cohomology of Lie algebras
  \item 17B66 Lie algebras of vector fields and related algebras
\end{itemize}

Zbl 1128.17008

\textbf{Karaali, Gizem}

\textbf{Constructing R-matrices on simple Lie superalgebras.} (English)
http://dx.doi.org/10.1016/j.jalgebra.2004.07.005
http://www.sciencedirect.com/science/journal/00218693

In this paper, \( R \)-matrices are constructed for simple classical Lie superalgebras with nondegenerate Killing form, thus extending the classical Lie-algebraic work of Belavin and Drinfeld.

\textit{Pasha Zusmanovich (Reykjavik)}

\textit{Keywords} : Lie superalgebra; \( R \)-matrix

\textit{Classification} :

\begin{itemize}
  \item 17B20 Simple and semisimple Lie algebras
  \item 17B62 Lie bialgebras
\end{itemize}

Zbl 1106.37038

\textbf{Skrypnyk, T.}

\textbf{Deformations of loop algebras and classical integrable systems: finite-dimensional Hamiltonian systems.} (English)
http://dx.doi.org/10.1142/S0129055X04002187
http://www.worldscinet.com/rmp/rmp.shtml

The author generalizes his previous construction of a family of quasi-graded (in the Krichever-Novikov sense) Lie algebras which are deformations of loop algebras, and develops a theory of certain finite-dimensional Hamiltonian systems associated with them. A lot of concrete examples of such Hamiltonian systems are worked out in detail. The Lie algebras considered are highly interesting: they are defined on the vector space
$g \otimes \mathbb{C}[t, t^{-1}]$ with bracket

$$[X \otimes f(t), Y \otimes g(t)] = [X, Y] \otimes f(t)g(t) - (XAY - YAX) \otimes tf(t)g(t)$$

where $g$ is one of the classical Lie algebras of type $gl(n)$, $so(n)$ or $sp(n)$, and $A$ is a suitably chosen matrix. Note that the bracket $XAY - YAX$ appearing as a tensor factor in the “deformation part”, defines, in its turn, a Lie algebra structure which already appeared in the context of Hamiltonian systems (see, e.g., [V. V. Trofimov and A. T. Fomenko, Algebra and geometry of integrable Hamiltonian differential equations, Faktorial, Moscow, Gorki: Prosperus (1995; Zbl 0858.58026)].

These algebras deserve, in the reviewer’s opinion, a further study of their properties from an algebraic viewpoint. It would be also interesting to compare them with another family of deformations of loop algebras in the class of Krichever-Novikov algebras constructed by A. Fialowski and M. Schlichenmaier [Global geometric deformations of current algebras as Krichever-Novikov type algebras, Commun. Math. Phys. 260, No. 3, 579–612 (2005; Zbl 1136.17307), see also arXiv:math.QA/0412113].

Pasha Zusmanovich (Reykjavik)

Keywords: finite-dimensional Hamiltonian system; deformation of loop algebra; quasi-graded Lie algebra

Classification:

* 37J35 Completely integrable systems, etc.
17B65 Infinite-dimensional Lie algebras
53D20 Momentum maps; symplectic reduction
70H06 Completely integrable systems and methods of integration

Zbl 1066.17014

Schlichenmaier, Martin; Sheinman, O.K.
The Knizhnik-Zamolodchikov equations for positive genus, and Krichever-Novikov algebras. (English. Russian original)
http://dx.doi.org/10.1070/RM2004v059n04ABEH000760
http://www.turpion.org/php/homes/pa.phtml?jrnid=rm
http://www.iop.org/EJ/journal/0036-0279

This is a big semi-survey paper continuing another one [Russ. Math. Surv. 54, 213–249 (1999); translation from Usp. Mat. Nauk 54, No. 1, 213–250 (1999; Zbl 0943.17019)] by the same authors, where they combine the survey of many aspects pertaining Krichever-Novikov algebras with a full exposition of new results.

The main question considered is: how can the Knizhnik-Zamolodchikov equations be generalized to a Riemann surface with a few punctures? The use of Krichever-Novikov algebras (as opposed to loop and Virasoro algebras) is a new ingredient the authors contribute to the question.

The main results are: construction of a generalized Knizhnik-Zamolodchikov connection on the conformal block bundle on an open dense part of the moduli space of curves with
marked points, and the proof of projective flatness of this connection. The preprint of the paper (under a different title) is available as arXiv:math.AG/0312040. However, the published version, unlike the preprint one, contains a very well-written introduction outlining the development of the question in the chronological order.

Pasha Zusmanovich (Ramat Gan)

Keywords: Krichever-Novikov algebras; Knizhnik-Zamolodchikov equations; conformal field theory

Classification:

*17B65 Infinite-dimensional Lie algebras
17-02 Research monographs (nonassoc. rings and algebras)
17B10 Representations of Lie algebras, algebraic theory
17B55 Homological methods in theory of Lie algebras
17B67 Kac-Moody algebras
17B68 Virasoro and related algebras
17B70 Graded Lie algebras
32G34 Moduli and deformations for ODE
53C07 Special connections and metrics on vector bundles
53C20 Riemannian manifolds (global)
81R10 Repres. of infinite-dim. groups and algebras from quantum theory
81T40 Two-dimensional field theories, etc.

Zbl 1066.17010

Kim, Yunhyong

The Lie algebra cohomology of jets. (English)
emis:journals/JLT/vol.14_no.1/11.html
http://www.heldermann.de/JLT/jltcover.htm
http://www.emis.de/journals/JLT/

In the classical paper [Ann. Math. (2) 74, 329–387 (1961; Zbl 0134.03501)], B. Kostant calculated the cohomology of a nilradical of a Borel subalgebra of a complex semisimple Lie algebra $\mathfrak{g}$ with coefficients in various modules. In [Invent. Math. 34, 37–76 (1976; Zbl 0358.17015)], H. Garland and J. Lepowsky generalized Kostant’s results to certain classes of infinite dimensional Lie algebras closely related to Kac-Moody algebras. Here the author reproves one of the results which follows from Garland-Lepowsky calculations, namely, she determines the (continuous) cohomology with trivial coefficients of a complex Lie algebra $\mathfrak{g} \otimes \mathbb{C}[[z]]$.

The method employed by the author is closer to the original Kostant’s one and consists of defining Laplace-like operator on a complex of semi-infinite forms, of which the standard Eilenberg-Chevalley complex calculating the cohomology of interest, is shown to be a subcomplex. This makes an interesting connection with semi-infinite cohomology.

Pasha Zusmanovich (Ramat Gan)

Keywords: cohomology of current Lie algebras; semi-infinite cohomology
Consider an associative algebra of differential operators in \(n\) indeterminates (with smooth or polynomial coefficients) with respect to composition. Its subspace \(W(n)\) of vector fields (i.e. first-order differential operators) constitutes a famous Lie algebra of general Cartan type with respect to commutator. This fact can be reformulated by saying that \(W(n)\) is closed under \(s_2\), where

\[
s_k(t_1, \ldots, t_k) = \sum_{\sigma \in S(k)} (-1)^{\sigma} t_{\sigma(1)} \cdots t_{\sigma(n)}
\]

is a skew-symmetrization operator.

The question considered in this paper is for which values of \(k\), \(W(n)\) (as well as other Lie algebras of Cartan type) is closed under \(s_k\).

The author conjectures this is true if and only if \(k = n^2 + 2n - 2\), proves the “if” part and provides many particular calculations supporting the “only if” part of the conjecture. It turns out that sometimes it is more convenient to represent \(s_k\) and related expressions in terms of the right-symmetric product on \(W(n)\) defined as

\[
u \partial_i \circ v \partial_j = v \partial_j(u) \partial_i.
\]

The paper is loaded with heavy calculations which are presented in an ordered and organized manner, and vividly illustrate the interplay of different topics: \(n\)-Lie algebras, identities and cohomology of Lie algebras of Cartan type, right-symmetric algebras.

The author was aided in his work by computer experiments on which he unfortunately not elaborates.

This is an interesting and thought-provoking paper.

A preprint version of this paper is available as arXiv:math.RA/0203036.

Pasha Zusmanovich (Amsterdam)

Keywords : differential operator; commutator; skew-symmetrizer; Lie algebras of Cartan type

Classification :

* 17B66 Lie algebras of vector fields and related algebras
This is an exposition of a talk based on the highly interesting work of the author and M. Schlichenmaier [Commun. Contemp. Math. 5, 921–945 (2003; Zbl 1052.17011); see also arXiv:math.QA/0206114], where the author continues her long-standing investigation of deformations of Lie algebras and related structures, combining particular calculations with philosophical insight, particularly, trying to answer the question “What are deformations and what are they good for?”

First, the author briefly revises the notions of infinitesimal, global (or geometric), formal and versal deformations of Lie algebras.

Then, she exhibits the situation when an algebra is rigid formally but not globally. Namely, she constructs a family of non-isomorphic Krichever-Novikov-type algebras containing a famous infinite dimensional Witt algebra as a member. This shows that the Witt algebra is not globally rigid. At the same time, it is well known that its second cohomology in an adjoint module is zero what implies its formal rigidity.

The author concludes that “the theory of deformations of infinite dimensional Lie algebras is far from satisfactory”.

**Pasha Zusmanovich (Amsterdam)**

**Keywords**: Krichever-Novikov algebras; Witt algebra; deformations; rigidity

**Classification**:

- **17B66** Lie algebras of vector fields and related algebras
- **14D15** (Formal) deformations
- **14H52** Elliptic curves
- **14H55** Riemann surfaces
- **17B55** Homological methods in theory of Lie algebras
- **17B68** Virasoro and related algebras

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**Zbl 1038.17017**

**Agrebaoui, B.; Ammar, F.; Lecomte, P.**

On the cohomology of the spaces of differential operators acting on skewsymmetric tensor fields or on forms, as modules of the Lie algebra of vector fields.
The first cohomology group of the Lie algebra of vector fields of a smooth manifold $M$ with coefficients in the space of linear differential operators acting on contravariant skewsymmetric tensor fields, or on differential forms of the manifold, is computed. The authors have in mind to study deformations of these modules (which are, via classical deformation theory, linked with the appropriate first cohomology group).

As it is often happens in such kinds of questions, the one-dimensional case $M = \mathbb{R}$ is special and not treated here.

\textit{Pasha Zusmanovich (Amsterdam)}

\textit{Keywords} : Lie algebra of vector fields; module of linear differential operators; first cohomology group; cohomology; differentiable manifold; differential operators

\textit{Classification} :

* 17B66 Lie algebras of vector fields and related algebras
17B56 Cohomology of Lie algebras
53C65 Integral geometry

\textbf{Zbl 1106.17039}

\textbf{Milas, Antun}

Correlation functions, differential operators and vertex operator algebras. (English)


This is an exposition of the work which appeared in detailed form in [\textit{A. Milas, J. Pure Appl. Algebra} 183, No. 1-3, 129–190, 191–244 (2003; Zbl 1036.17019)].

The subject of study is representation theory of Lie superalgebras of differential operators on the circle, with diversions to different areas of mathematics and theoretical physics.

Vertex operator superalgebras structures appear on the respective representations spaces, and values of the $\zeta$-function appear as normalizing terms of certain vertex operators.

The author generalizes to the supercase earlier constructions of other authors, and introduces a new class of correlation functions that governs generalized characters, gradation and filtration of the underlying Lie superalgebra and their representations.

This is a difficult and sophisticated mathematics.

\textit{Pasha Zusmanovich (Reykjavik)}

\textit{Keywords} : Lie superalgebras of differential operators; Riemann zeta-function; vertex operator algebras; correlation functions
Grozman, Pavel; Leites, Dimitry; Shchepochkina, Irina
Defining relations for the exceptional Lie superalgebras of vector fields. (English)

This is a paper in a long and interesting series of papers by Leites and his collaborators devoted to classification and structural properties of simple and related to them Lie superalgebras.

The subject of the paper are 5 exceptional Lie superalgebras of vector fields with polynomial coefficients, whose regradings give all known 15 exceptional simple vectorial Lie superalgebras.

The main result is a description of the defining relations for locally nilpotent Lie superalgebras $L_-$ and $L_+$ (the latter is a sum of the positive part in the grading of the whole algebra plus $(L_0)_+$, the positive part in the grading of the zero component) in the respective standard gradings of those exceptional Lie superalgebras.

The approach is to interpret defining relations as the second homology, and use the Hochschild-Serre spectral sequence with respect to $(L_0)_+$.

The authors were assisted by the Mathematica-based package SuperLie developed by one of the authors.

Somewhat surprisingly, the cases of polynomial vector fields and Laurent polynomials ones differ drastically (the latter is described in other papers of the authors).

As the other papers in the series, it is very thoroughly written: an introduction to linear super-algebras is given, the relationship to homological algebras is explained, and a description of simple vectorial Lie superalgebras is presented. It could serve as a concise introduction to the subject of Lie superalgebras (which occupies about half of the paper).

Pasha Zusmanovich (Ramat Gan)

Keywords : exceptional Lie superalgebras of polynomial vector fields; defining relations

Classification : 
*17B66 Lie algebras of vector fields and related algebras
17-08 Computational methods
17B55 Homological methods in theory of Lie algebras
17B56 Cohomology of Lie algebras
Consider embedding of a Lie superalgebra $K(1)$ of contact vector fields into a Lie superalgebra of superdifferential operators on the supercircle $S^{1|1}$. The authors calculate the first cohomology if $K(1)$ with coefficients in the corresponding $K(1)$-module, as well as in the module of tensor densities.

*Pasha Zusmanovich (Amsterdam)*

**Keywords**: Lie superalgebra of vector fields; tensor densities

**Classification**:
- *17B56* Cohomology of Lie algebras
- *17B66* Lie algebras of vector fields and related algebras

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A variant of this paper has already been reviewed [A. Fialowski, Lect. Notes Pure Appl. Math. 236, 177–188 (2004; Zbl 1048.17010)].

It is shown that an infinite dimensional Witt algebra can be included in a parametric family of Krichever-Novikov algebras, therefore it is not globally rigid. This contrasts with a known fact of the formal rigidity of the Witt algebra.

In addition to the material presented in the above-mentioned paper, the authors consider how the subalgebra $L_1$ of the Witt algebra fits the picture.

*Pasha Zusmanovich (Amsterdam)*

**Keywords**: Krichever-Novikov algebras; Witt algebra; deformations; rigidity

**Classification**:
- *17B66* Lie algebras of vector fields and related algebras
- *14D15* (Formal) deformations
- *14H52* Elliptic curves
- *14H55* Riemann surfaces
- *17B55* Homological methods in theory of Lie algebras
- *17B68* Virasoro and related algebras
These are the first two parts of the three-paper series based on the author’s Ph.D. thesis, where he elucidates the relationships between certain spinor representations of Lie (super)algebras of (pseudo)differential operators on the circle (among them the celebrated Virasoro and Neveu-Schwarz algebras), values of the Riemann zeta-function at the negative integers, vertex operator algebras and $q$-difference equations.


Pasha Zusmanovich (Amsterdam)

**Keywords**: Lie superalgebras of differential operators; Riemann zeta-function; vertex operator algebras; $q$-difference equations; n-point correlation functions; graded $q$-traces; elliptic transformation

**Classification**:

$\ast$17B66 Lie algebras of vector fields and related algebras
17B69 Vertex operators
11M06 Riemannian zeta-function and Dirichlet L-function

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An element of a Weyl algebra is called strictly nilpotent if it acts Lie-nilpotently on the whole algebra. The author gives some characterization of strictly nilpotent elements of the first Weyl algebra $A_1$ and deduces from it, first, certain characterization of bispectral operators (with polynomial coefficients) and, second, a reformulation of the Dixmier-Kirillov conjecture – that every endomorphism of $A_1$ is an automorphism – in terms of bispectral operators.
This construction could be extended to the Lie (super)algebra of Hamiltonian vector fields on the odd space, yielding another graph-complex (it seems that the preprint of Kontsevich and Shoikhet the author is referring to, does not exist, at least at the time of writing of this review).
In this paper, it is shown, in a purely combinatorical way, that the cohomology of these two graph-complexes coincide, which yields coincidence of cohomology of corresponding Lie (super)algebras.
It would be interesting to find a direct proof of the latter fact not appealing to graph-complexes, and, according to Feigin (quoted by the author), this seems to be a difficult question.

Pasha Zusmanovich (Ramat Gan)

Keywords: Lie algebra of Hamiltonian vector fields; graph-complex

Classification:

*17B66 Lie algebras of vector fields and related algebras
05C99 Graph theory
17B56 Cohomology of Lie algebras
18G35 Chain complexes (homological algebra)
There are quite a few results in the literature about embeddings of Lie algebras into Lie algebras of vector fields or their relatives. One of the most remarkable among them is the “realization theorem” of V. Guillemin and S. Sternberg [Bull. Am. Math. Soc. 70, 16–47 (1964; Zbl 0121.38801)], whose constructive version was given by R. J. Blattner [Trans. Am. Math. Soc. 144, 457–474 (1969; Zbl 0295.17002)]. It says, roughly, that a Lie algebra $L$ with a subalgebra $L_0$ of finite codimension $n$ satisfying some additional finitness conditions, embeds into the Lie algebra of formal vector fields $W_n$ in such a way that $L_0$ embeds into the zero component of $W_n$ relative to its standard filtration. Such embedding is called a transitive realization. Depending on the underlying ring of vector fields coefficients – polynomial, formal, analytic, etc., the transitive realization is called respectively polynomial, formal, analytic, etc. So, the Guillemin-Sternberg-Blattner theorem asserts, particularly, that any finite-dimensional pair $L_0 \subset L$ has a formal realization.

In characteristic $p$, there is a whole array of similar results belonging to various authors [see H. Strade, The classification of the simple Lie algebras over fields with positive characteristic, Hamb. Beitr. Math. 31, S2.4–S2.6 (1997)] and references therein. On the other hand, there are (characteristic 0) examples of finite-dimensional pairs $L_0 \subset L$ not having a polynomial transitive realization, what justifies the desire to search for realizations with coefficients larger than polynomials and smaller than formal power series.

On this way, the author considers the following conjecture: every finite-dimensional pair has a realization with coefficients in a subalgebra of formal power series whose elements satisfy a linear ordinary differential equation with constant coefficients. One may observe that over algebraically closed field, the latter condition is equivalent to those of being generated by polynomials and exponents.

This, in its turn, allows the author to show that his conjecture is essentially equivalent to a problem formulated by Sophus Lie in terms of transitive groups acting on an $n$-dimensional Euclidean space.

Some particular cases of the conjecture are proved, namely, for the case when $L$ is decomposed (as a vector space) as a direct sum of two subalgebras: $L = L_0 \oplus Q$, and $Q$ acts nilpotently on $L$ (in fact in this case one achieves a realization even with polynomial coefficients), when there is a chain of subalgebras of codimension 1 between $L_0$ and $L$, and when the codimension of $L_0 \leq 3$.

In conclusion, the author presents two examples of GAP sessions computing polynomial realizations using his own package justly named Blattner.

**Pasha Zusmanovich (Amsterdam)**

**Keywords**: embedding into Lie algebras of vector fields; transitive realization; polynomial and formal coefficients; transitive group action; Sophus Lie; GAP

**Classification**:

*17B66* Lie algebras of vector fields and related algebras

17-04 Machine computation, programs (nonassoc. rings and algebras)
The subject of this interesting paper are deformations of the module of symbols (= symmetric contravariant tensor fields, = polynomial functions on the space of tensors) over a Lie algebra $\text{Vect} (\mathbb{R}^n)$ of smooth vector fields on $\mathbb{R}^n$. The module structure is defined (in the local coordinates) by the formula for the Hamiltonian vector field.

The authors consider miniversal deformations in the spirit of A. Fialowski [in “Deformation Theory of Algebras and Structures and Applications”, Kluwer, NATO ASI Ser., Ser. C 247, 375-401 (1988; Zbl 0663.17009)] which are, in a sense, universal objects in the category of all deformations with a base in a commutative ring. Particularly, multi-parameter deformations are treated.

The authors determine all miniversal differential deformations (that is, deformations whose terms are all given by differentiable functions) and put a noticeable example of the $\text{Vect} (\mathbb{R}^n)$-module of linear differential operators into a deformation-theoretic context.

According to the general principle, the deformations of a module $V$ over a Lie algebra $L$ are controlled by the low-dimensional cohomology $H^*(L, \text{End}(V))$. The relevant first cohomology group (restricted by considerations of only differentiable cochains) was determined by P. Lecomte and V. Ovsienko [Compos. Math. 124, 95-110 (2000; Zbl 0968.17007)], and the authors give criteria for integrability of the corresponding cocycles.

The important ingredient in the proofs is invariance of all relevant deformations under an action of some distinguished subalgebras lying in the zero- and $-1$-components with respect to the standard grading of $\text{Vect} (\mathbb{R}^n)$.

Pasha Zusmanovich (Amsterdam)

*Keywords* : multiparameter deformations; miniversal deformations; algebraic deformation theory; Lie algebra of vector fields; linear differential operators

*Classification* :

* 17B66 Lie algebras of vector fields and related algebras
* 17B56 Cohomology of Lie algebras
* 17B70 Graded Lie algebras
A groupoid is called a pogroupoid if for all its elements $x, y, z$ the following three conditions are satisfied: $xy \in \{x, y\}$, $x(yx) = yx$ and $(xy)(yz) = (xy)z$. A pogroupoid can be turned into a partially ordered set (poset) by letting $x \leq y$ iff $yx = y$. These notions were introduced by J. Neggers [Kyungpook Math. J. 16, 7-20 (1976; Zbl 0355.06004)]. One may define a pogroupoid algebra analogously to a (semi)group algebra. The parallel dimension of a pogroupoid is the dimension of the Lie algebra associated to its pogroupoid algebra.

In the paper under review, the notion of parallel dimension is studied and its relation to the properties of the poset associated to an initial pogroupoid is established.

Pasha Zusmanovich (Amsterdam)

Keywords: pogroupoid; poset; parallel dimension

Classification:
- 06A06 Partial order
- 20N02 Sets with a single binary operation (groupoids)
- 06A11 Algebraic aspects of posets
- 16S36 Ordinary and skew polynomial rings and semigroup rings
- 17B60 Lie rings associated with other structures

Zbl 1010.33006

Airault, Hélène; Ren, Jiagang

An algebra of differential operators and generating functions on the set of univalent functions. (English)


http://dx.doi.org/10.1016/S0007-4497(02)01115-6

http://www.sciencedirect.com/science/journal/00074497


The context is representations of the Virasoro algebra.

Pasha Zusmanovich (Amsterdam)

Keywords: Faber polynomials; univalent functions; vector fields; Virasoro algebra

Classification:
- 33C80 Connections of theory of special functions with groups and algebras
- 30C55 General theory of univalent and multivalent functions
- 17B68 Virasoro and related algebras
- 35A30 Geometric theory for PDE, transformations
- 32M25 Complex vector fields

Zbl 1009.17017

Boyallian, Carina; Liberati, Jose I.

On modules over matrix quantum pseudo-differential operators. (English)
Consider the following (commutative) diagram of Lie algebras, where horizontal arrows denote passing to an algebra of matrices and vertical ones denote quantization:

\[
\begin{array}{ccc}
\hat{D} & \longrightarrow & \hat{D}^M \\
\downarrow & & \downarrow \\
\hat{S}^q & \longrightarrow & \hat{S}^q_M
\end{array}
\]

Here \( \hat{D} \) is the central extension of differential operators on the circle, \( \hat{S}^q \) is the central extension of quantum pseudo-differential operators on the circle, \( \hat{D}^M \) is the central extension of \( M \times M \) matrix differential operators on the circle, and \( \hat{S}^q_M \) is the central extension of \( M \times M \) matrix quantum pseudo-differential operators on the circle. The quasifinite highest-weight modules for these algebras were described by V. Kac and A. Radul in [Commun. Math. Phys. 157, 429-457 (1993; Zbl 0826.17027)] for the cases of \( \hat{D} \) and \( \hat{S}^q \) and by C. Boyallian, V. Kac, J. Liberati, and C. Yan in [J. Math. Phys. 39, 2910-2928 (1998; Zbl 0999.17032)] for the case of \( \hat{D}^M \). Here the authors make a next logical step by doing the same for \( \hat{S}^q_M \). Both the results and methods are very similar to those in the cited works.

Pasha Zusmanovich (Amsterdam)

Keywords: quantum pseudo-differential operators; central extension; highest-weight modules

Classification:

- \*17B66 Lie algebras of vector fields and related algebras
- 17B10 Representations of Lie algebras, algebraic theory
- 81R10 Repres. of infinite-dim. groups and algebras from quantum theory

Zbl 0995.17009

Larsson, Anna

A periodisation of semisimple Lie algebras. (English)
Homology Homotopy Appl. 4, No.2(2), 337-355, electronic only (2002). ISSN 1532-0073; ISSN 1532-0081
emis:journals/HHA/volumes/2002/volume4-2-2.htm
http://intlpress.com/hha/
http://projecteuclid.org/hha
http://www.emis.de/journals/HHA/

A periodisation of a Lie algebra \( L \) over a field \( K \) is a Lie algebra \( L \otimes tK[t] \) with a Lie bracket defined naturally by tensor components. (So, roughly speaking, periodisation is a positively-graded Lie algebra with an underlying Lie algebra in each degree.)
In this interesting paper the author proves that a periodisation of a semisimple Lie algebra over an algebraically closed field of characteristic 0 without direct summands isomorphic to $\mathfrak{sl}(2)$ possesses a presentation with only quadratic relations. Such a presentation is explicitly constructed in terms of a Chevalley basis of an underlying Lie algebra.


Pasha Zusmanovich (Amsterdam)

Keywords: graded Lie algebras; Chevalley basis; current algebras; presentations; periodisation

Classification:

* 17B70 Graded Lie algebras
  17B20 Simple and semisimple Lie algebras

Zbl 0995.17010

Grozman, P.; Leites, D.; Poletaeva, E.

Defining relations for classical Lie superalgebras without Cartan matrices. (English)

Homology Homotopy Appl. 4, No.2(2), 259-275, electronic only (2002). ISSN 1532-0073; ISSN 1532-0081

emis:journals/HHA/volumes/2002/volume4-2-2.htm
http://intlpress.com/hha/
http://projecteuclid.org/hha
http://www.emis.de/journals/HHA/

From the abstract: “The analogs of Chevalley generators are offered for simple (and close to them) Z-graded complex Lie algebras and Lie superalgebras of polynomial growth without Cartan matrix. We show how to derive the defining relations between these generators and explicitly write them for a “most natural” system of simple roots.”

As usual in works of Leites and his collaborators, things are put into historical perspective and presented as a part of the broad “supermathematics” program, and open problems together with possible ways to approach them are outlined.

Pasha Zusmanovich (Amsterdam)

Keywords: graded Lie superalgebras; defining relations

Classification:

* 17B70 Graded Lie algebras
  17B01 Identities, free Lie algebras
  17B20 Simple and semisimple Lie algebras
  17B65 Infinite-dimensional Lie algebras
  17B66 Lie algebras of vector fields and related algebras
  17B67 Kac-Moody algebras
As the title suggests, this interesting paper deals with representations of the (infinite-dimensional, “two-sided”) Virasoro algebra. The representations considered are bounded, i.e. decomposable as a direct sum of finite-dimensional eigenspaces with respect to $e_0$-action, and dimensions of eigenspaces are uniformly bounded. Such representations of length 1 (i.e. irreducible) and length 2 were classified by O. Mathieu [Invent. Math. 107, 225–234 (1992; Zbl 0779.17025)] and C. Martin and A. Piard [Commun. Math. Phys. 137, 109–132 (1991; Zbl 0728.17015) and Commun. Math. Phys. 150, 465–493 (1992; Zbl 0774.17036)].

Here the author makes a next step by classifying indecomposable generic bounded representations of length 3 (“generic” means that $e_0^2 + e_0 - e_{-1}e_1$, the Casimir operator of the zero term in the standard grading, acts on composition series with pairwise distinct eigenvalues; the non-generic case is promised to be considered in a future paper). He also provides a new classification in the length 2 case.

The main computational tool is a certain interesting (though straightforward) cohomological interpretation of module extensions in terms of cup-products. The author treats it as folklore (without providing a reference), while, in reviewer’s opinion, it is not. Although the most of the proofs are done by laborous computations, the author often outlines possible ways for a more “conceptual” alternative proofs.

Both length 2 and length 3 representations occur to be exactly restrictions of the corresponding representations of the distinguished “one-sided” subalgebra $\langle e_{-1}, e_0, e_1, e_2, \ldots \rangle$. The author admits that he has no conceptual explanation of this fact.

The paper concludes with observations and conjectures about the cases of higher length.

Pasha Zusmanovich (Amsterdam)

Keywords: Virasoro algebra; indecomposable modules

Classification:

* 17B68 Virasoro and related algebras
  17B10 Representations of Lie algebras, algebraic theory
  17B55 Homological methods in theory of Lie algebras

Wagemann, Friedrich

Explicit formulae for cocycles of holomorphic vector fields with values in $\lambda$ densities. (English)
J. Lie Theory 11, No. 1, 173-184 (2001). ISSN 0949-5932
http://www.heldermann.de/JLT/jltcover.htm
http://www.emis.de/journals/JLT/
The subject of this paper is cohomology of a Lie algebra of holomorphic vector fields on a punctured Riemann surface $\Sigma_r$, with coefficients in a module of holomorphic $\lambda$-densities on $\Sigma_r$.

Results about this cohomology were obtained by N. Kawazumi [Ann. Inst. Fourier 43, 655–712 (1993; Zbl 0782.57019)], but explicit cocycles were not known. Here they are provided for the second cohomology.

This is motivated by a search for generalizations of Krichever-Novikov algebras as corresponding abelian extensions, analogous to the generalization of Virasoro algebras by V. Ovsienko and C. Roger [Indag. Math. 9, 277–288 (1998; Zbl 0932.17029)].

Pasha Zusmanovich (Amsterdam)

Keywords: Lie algebra of vector fields; densities; 2-cocycles; punctured Riemann surface; Krichever-Novikov algebras

Classification:  
- 17B56 Cohomology of Lie algebras  
- 17B66 Lie algebras of vector fields and related algebras  
- 30F30 Differentials on Riemann surfaces

Zbl 1036.17018

Gargoubi, H.; Ovsienko, V.  
Modules of differential operators on the real line. (English. Russian original)  
http://dx.doi.org/10.1023/A:1004116431929  
http://www.springerlink.com/content/106470/

Let $\text{Vect}(M)$ be a Lie algebra of vector fields on a smooth manifold $M$. The authors consider a family of $\text{Vect}(M)$-modules of $k$th order differential operators, depending on two parameters $\lambda, \mu \in \mathbb{R}$, defined on the space $\text{Hom}(F_\lambda, F_\mu)$, where $F_\lambda$ is a module of tensor densities of degree $-\lambda$.

The question of classification of these modules up to isomorphism was solved earlier in the case $\dim M > 1$ [P. B. A. Lecomte, P. Mathonet and E. Tousset, Indag. Math., New Ser. 7, 461–471 (1996; Zbl 0892.58002) and P. Mathonet, Commun. Algebra 27, 755–776 (1999; Zbl 0924.17017)]. Here the authors solve this question for the one-dimensional case $M = \mathbb{R}$, which proves to be exceptional and apparently more difficult than the general case.

The proof employs, among other the fact that these modules are filtered deformations of modules of symbols of differential operators, defined via particular cocycles from the relevant first-order cohomology group.

In many cases, the proofs and computations are omitted or only sketched, and details are promised to be published elsewhere.

Pasha Zusmanovich (Amsterdam)
This is a survey of some notions and results related to the cohomology of Lie algebras, motivated by or having applications in physics. Though being far from describing the subject fully, it gives a nice interesting account of some particular topics. The authors start by introducing all needed notions – mainly cohomology of Lie algebras – making therefore the article self-contained. Though probably they formally achieve this goal, the reviewer finds this part of exposition somewhat cumbersome (or, put it another way, “physics-inclined”), both in notations (big amount of tensor-calculus-like sub- and super-scripts) and accuracy (for example, one may guess that the ground field is of characteristic zero or even \( \mathbb{C} \) or \( \mathbb{R} \), though it is never stated explicitly).

Probably the most interesting (in the reviewer’s opinion) is the exposition of the results, due to the authors, about the higher-order Lie algebra structures related to classical simple Lie algebras and their cohomology. Other topics include de Rham cohomology, BRST formalism, strong homotopy algebras and higher-order Poisson structures.

As each survey that is worth its name, it concludes with a large bibliography.

Pasha Zusmanovich (Amsterdam)

Keywords: Lie algebra cohomology; classical simple Lie algebra; higher-order Lie algebra; higher-order Poisson structure; BRST; survey; applications in physics; bibliography

Classification:

* 17B56 Cohomology of Lie algebras
17-02 Research monographs (nonassoc. rings and algebras)
81T70 Quantization in field theory; cohomological methods
17B63 Poisson algebras
17B81 Applications of Lie algebras to physics
14F40 De Rham cohomology
17B20 Simple and semisimple Lie algebras
17A42 Other n-ary compositions
Poncin, Norbert
On the cohomology of the Nijenhuis-Richardson graded Lie algebra of the space of functions of a manifold. (English)
http://dx.doi.org/10.1006/jabr.2001.8827
http://www.sciencedirect.com/science/journal/00218693

This is a full proof of results presented by the author in [Bull. Belg. Math. Soc. – Simon Stevin 8, No. 1, 141-146 (2001; Zbl 0989.17012)] about low-dimensional cohomology of the Nijenhuis-Richardson graded Lie algebra of the space of functions on a manifold, with coefficients in an adjoint module.

Pasha Zusmanovich (Amsterdam)
Keywords : Nijenhuis-Richardson Lie algebra; Lie algebra cohomology; de Rham cohomology
Classification :
* 17B56 Cohomology of Lie algebras
  17B66 Lie algebras of vector fields and related algebras

Zbl 0989.17012

Poncin, Norbert
First cohomology spaces of the Nijenhuis-Richardson graded Lie algebra of the space of functions of a manifold. (Premiers espaces de la cohomologie de l’algèbre de Lie graduée de Nijenhuis-Richardson de l’espace des fonctions d’une variété.) (French)
http://www.emis.de/journals/BBMS/bms.bull.html
http://projecteuclid.org/DPubS?service=UIversion=1.0verb=Displayhandle=euclid.bbms

In this paper, results about the low-dimensional cohomology of the Nijenhuis-Richardson graded Lie algebra of the space of functions of a manifold, with coefficients in an adjoint module, are presented. This continues an earlier work of the author [C. R. Acad. Sci., Paris, Sér. I 328, 789-794 (1999; Zbl 0968.17008)].
No detailed proofs are given.

Pasha Zusmanovich (Amsterdam)
Keywords : Nijenhuis-Richardson Lie algebra; Lie algebra cohomology; de Rham cohomology
Classification :
* 17B56 Cohomology of Lie algebras
  17B66 Lie algebras of vector fields and related algebras

47
Zbl 1024.17016

Bouarroudj, S.; Ovsienko, V.Yu.

Schwarzian derivative related to modules of differential operators on a locally projective manifold. (English)


http://journals.impan.gov.pl/bc/

The authors introduce a 1-cocycle on the group Diff(M) of diffeomorphisms of a smooth manifold M with coefficients in the module of differential operators with values in Hom(S^2, C^\infty(M)).

The case M = S^1 was considered by authors earlier in [Int. Math. Res. Not. 1998, 25-39 (1998; Zbl 0919.57026)]. In this case the cocycle coincides with the classical Schwarzian derivative, so the general case considered here may be treated as a (yet another) generalization of the Schwarzian derivative (other previously existing generalizations are briefly reviewed in the article).

The authors also discuss the interesting problem of a full description of the relevant (first-order) cohomology group (in the case M = S^1 it is one-dimensional, being exhausted by the above-mentioned cocycle).

Pasha Zusmanovich (Amsterdam)

Keywords: cohomology of group of diffeomorphisms; Schwarzian derivative

Classification:

*17B56 Cohomology of Lie algebras
57S25 Groups acting on specific manifolds
81T70 Quantization in field theory; cohomological methods
17B66 Lie algebras of vector fields and related algebras
58D05 Groups of diffeomorphisms and homeomorphisms as manifolds

Zbl 1003.17010

Zharinov, V.V.

On the cohomology of the Heisenberg algebra. (English. Russian original)

A Heisenberg Lie algebra is understood here to be a 1-dimensional central extension of an abelian Lie algebra (of arbitrary dimension).

Cohomologies of Heisenberg Lie algebras are studied and in some cases are computed explicitly. Remark: The use of Hochschild-Serre spectral sequence would simplify the part of the author’s calculations.

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Keywords: Heisenberg algebra; cohomology
Classification:

* 17B56 Cohomology of Lie algebras
  17B55 Homological methods in theory of Lie algebras